MATH*1200 – LAB #4 SOLUTION

Solution to: www.nathanieljohnston.com/math1200/lab4.pdf

To show that $h(x)$ is continuous, we must shows that

$$\lim_{x\to a^-} h(x) = \lim_{x\to a^+} h(x) = h(a)$$

for $a = 3, 4$, and 6. This is done by simply plugging numbers into the appropriate branches of $h(x)$.

$$\begin{aligned}
\lim_{x\to 3^-} h(x) &= 3 + 3 = 6 \\
\lim_{x\to 3^+} h(x) &= 9 - 3 = 6 \\
h(3) &= 9 - 3 = 6
\end{aligned}$$

$$\begin{aligned}
\lim_{x\to 4^-} h(x) &= 9 - 4 = 5 \\
\lim_{x\to 4^+} h(x) &= -4(4)^2 + 39(4) - 87 = -64 + 156 - 87 = 5 \\
h(4) &= -4(4)^2 + 39(4) - 87 = -64 + 156 - 87 = 5
\end{aligned}$$

$$\begin{aligned}
\lim_{x\to 6^-} h(x) &= -4(6)^2 + 39(6) - 87 = -144 + 234 - 87 = 3 \\
\lim_{x\to 6^+} h(x) &= -(6)^2 + 16(6) - 57 = -36 + 96 - 57 = 3 \\
h(6) &= -(6)^2 + 16(6) - 57 = -36 + 96 - 57 = 3
\end{aligned}$$

Thus, $h(x)$ is continuous because the left and right limits are all equal, and they equal the function value itself for each of $a = 3, 4$, and 6.

Department of Mathematics & Statistics, University of Guelph, Guelph, ON, Canada