MATH*1200 – LAB #2 SOLUTION

Solution to: www.nathanieljohnston.com/math1200/lab2.pdf

1. To check whether or not Batman catches up to the Joker, set their positions equal and solve for the time x:

$$\begin{aligned} \sqrt{x^2+6x-16} &= x \\ \implies x^2+6x-16 &= x^2 \\ \implies 6x-16 &= 0 \\ \implies 6x &= 16 \\ \implies x &= \frac{8}{3}. \end{aligned}$$

   Thus, Batman catches the Joker at time $x = \frac{8}{3}$ seconds. That's pretty quick considering he didn't even get moving until time $x = 2$ seconds!

2. We start out the exact same as with question 1 – set their positions equal and solve for the time x:

$$\begin{aligned} \sqrt{x^2-6x-16} &= x \\ \implies x^2-6x-16 &= x^2 \\ \implies -6x-16 &= 0 \\ \implies -6x &= 16 \\ \implies x &= -\frac{8}{3}. \end{aligned}$$

   However, having a negative time doesn't make any sense given the setup of the question (alternatively, we can see that this "answer" makes no sense by substituting it back into the original equation $\sqrt{x^2-6x-16} = x$ and seeing that $x = -\frac{8}{3}$ doesn't work – remember that when we square both sides of an equation, we might introduce "fake" solutions). Thus, Batman never catches up to the Joker. We must now find how close he gets to the Joker as $x \to \infty$.

The distance between the white van and the Batmobile at time x seconds is given by $x-\sqrt{x^2-6x-16}$. Now we compute the limit:

$$\begin{aligned}
\lim_{x\to\infty}\left(x-\sqrt{x^2-6x-16}\right) &= \lim_{x\to\infty}\left(x-\sqrt{x^2-6x-16}\cdot\frac{x+\sqrt{x^2-6x-16}}{x+\sqrt{x^2-6x-16}}\right)\\
&= \lim_{x\to\infty}\left(\frac{x^2-(x^2-6x-16)}{x+\sqrt{x^2-6x-16}}\right)\\
&= \lim_{x\to\infty}\left(\frac{6x+16}{x+\sqrt{x^2-6x-16}}\right)\\
&= \lim_{x\to\infty}\left(\frac{6+\frac{16}{x}}{1+\sqrt{1-\frac{6}{x}-\frac{16}{x^2}}}\right)\\
&= \frac{6+0}{1+\sqrt{1-0-0}}\\
&= 3.
\end{aligned}$$

Thus, as time rolls on and on, the Joker stays consistently about 3 metres ahead of the Batmobile.

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